Budget Balanced Mechanisms for the Multicast Pricing Problem with Rates*

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1. INTRODUCTION

Multicast transmissions allow huge savings of network traffic compared to unicast transmissions when the same data is sent to a lot of users. These savings are achieved by the fact that users may "share" links, since each node in a multicast network can send an incoming transmission to an arbitrary number of neighbours. If there are costs incurred when using an edge, then this sharing is an obstacle for pricing.

Formally, the (binary) multicast pricing problem is defined as follows: Let $G = (V, E)$ be an edge weighted undirected graph. The graph G models the underlying network, edge weight c_e represents the costs for using edge e. There is a distinguished set $N \subseteq V$ of users. Furthermore, there is a node $r \notin N$, the service provider. A cost-sharing mechanism determines which users receive the transmission and assigns a price to each of these users. Each user $i \in N$ has a (secret) utility u_i. He derives utility u_i from getting the transmission. If i gets the transmission at price x_i, his individual welfare is $u_i - x_i$. If i does not get the transmission, his welfare is $-x_i$. However, the cost-sharing mechanism does not a priori know the values u_i. It has to rely on the users to report these values. The users are selfish and thus might not be willing to report their true utility. In a game-theoretic framework, their set of strategies is to report any value $b_i \geq 0$ as their utility. Given these bids b_i, the task of the mechanism is to select a subset $Q \subseteq N$ of the users, find a multicast tree F serving Q, and assign prices x_i to the users. The cost-sharing mechanism for the tree F should meet some of the following socio-economic and game-theoretic properties: No Positive Transfer (NPT), Voluntary Participation (VP), Consumer Sovereignty (CS), Group Strategyproof (GSP) or Strategyproof (SP), Budget Balance (BB), and Efficiency (EFF). For a definition of these terms, see e.g. [1, 2, 4]. We also define these properties in Section 3 for rated problems, which include binary problems as a special case. It is a classical result in game theory that there is no strategyproof mechanism that meets both BB and EFF. From a computational point of view, we also want that the mechanism can efficiently be computed. In a distributed setting, it might also be desirable that the mechanism can be computed with low communication costs.

2. RELATED WORK

Most of the current pricing mechanisms for multicast transmissions assume that the underlying multicast tree is fixed, that is, G is a tree with root r and leaves N (see for instance [1, 2]). Thus for any subset of the users to be served, the tree used is a subtree of the underlying fixed tree. From the viewpoint of combinatorial optimization, this problem is not very interesting. For fixed trees, mechanisms are designed and analyzed that meet—beside NPT, VP, and CS—either GSP and BB or SP and EFF. The work of Jain and Vazirani [4] is a notable exception, as they do not assume that there is a fixed multicast tree.

Most of the pricing mechanisms mentioned above are binary, that means, either a user gets the full transmission or nothing at all. In a network with widely differing bandwidth connections—such as the internet—it is however unavoidable to have transmissions of data at different qualities or *rates*, say $\rho_1 \leq \rho_2 \leq \cdots \leq \rho_\ell$, where the number of rates ℓ is determined in advance. Adler and Rubenstein [1] proposed two approaches to handle different rates, which both reflect practice: Under the *layered* paradigm, the transmission is sent in layers. Layer 1 has rate ρ_1 and every other layer $i > 1$ has rate $\rho_i - \rho_{i-1}$. To receive rate ρ_j, a user is sent layers $1, \ldots, j$. Under the *split session* paradigm, there is a separate multicast transmission for each rate. Each user receives at most one of those transmissions. Adler and

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Rubenstein study *marginal cost* mechanisms under those paradigms. They assume that a fixed multicast tree is given. They do not treat budget balanced mechanisms or general graphs and pose those extensions as an open problem.

3. PROBLEMS WITH RATES

We here address the open problems posed by Adler and Rubenstein. We also propose two new paradigms (LC, SSC) for mechanisms with rates.

Now each user i has an utility vector $u_i = (u_{i,1}, \ldots, u_{i,\ell})$ and $u_{i,\lambda}$ is the utility of i when receiving the transmission at rate ρ_λ. The possible strategies of each user i is to bid a vector $b_i = (b_{i,1}, \ldots, b_{i,\ell})$, where $b_{i,\lambda} \geq 0$ indicates the price that i is willing to pay for rate ρ_λ. We are studying mechanisms that, given those n bids $b = (b_1, \ldots, b_n)$, compute a function $q : N \to \{0, \ldots, \ell\}$. (In the case of binary mechanisms, q simply is a characteristic function.) For each user i, $q_i := q(i)$ is the rate of the transmission received by i. $q_i = 0$ means that the user does not receive the transmission at all. Such a function q will be called a *rate function*. The mechanism also provides a function $x : N \to \mathbb{R}$. $x_i := x(i)$ denotes the price that user i has to pay to receive the transmission at rate ρ_{q_i}. The individual welfare of user i is $u_{i,q_i} - x_i$ provided that $q_i > 0$, since he gets the transmission at rate ρ_{q_i} for the price x_i. Otherwise, his welfare is $-x_i$. Finally, $\mathrm{Cost}(q)$ denotes the true costs incurred by the service provider when serving the users at rates according to the function q.

The properties NPT, VP, CS, (G)SP, and BB are refined as follows to handle multiple rates.

No Positive Transfer: For all users i, $x_i \geq 0$.

Voluntary Participation (VP): If $q_i > 0$, then $b_{i,q_i} - x_i \geq 0$, otherwise $x_i = 0$.

Consumer Sovereignty (CS): For every user i and for every rate ρ_λ, there is an ℓ-vector $\hat{b}_i^\lambda$ such that if i bids $\hat{b}_i^\lambda$, then i will get the service at rate ρ_λ (independent of the other bids).

Group Strategyproof (GSP): Even if a set of users C collude, their dominant strategy is to report their true utility u_i as b_i for all $i \in C$. If this property holds only for sets C of size one, then we speak of **Strategyproof (SP)**.

Budget Balance (BB): $\sum_{i\in N} x_i = \mathrm{Cost}(q)$, i.e., neither a deficit nor a surplus is created. If only $\mathrm{Cost}(q) \leq \sum_{i\in N} x_i \leq \alpha\cdot\mathrm{Cost}(q)$ holds, then we speak of **α-approximate Budget Balance (α-BB)**.

A binary mechanism for the multicast pricing problem can be interpreted as a rated mechanism with only one possible rate. Since such mechanisms are well studied, it is a natural design paradigm to construct rated mechanism for the multicast pricing problem from binary ones.

Let $M_1, M_2, \ldots, M_\ell$ be mechanisms for rates $\rho_1, \rho_2 - \rho_1, \ldots, \rho_\ell - \rho_{\ell-1}$ under the layered paradigm or for rates $\rho_1, \rho_2, \ldots, \rho_\ell$ under the split session paradigm, respectively, such that all of $M_1, \ldots, M_\ell$ meet NPT, VP, CS, GSP, and BB. Moulin [5] showed that for each such mechanism M_λ, there is a cross-monotonic cost-sharing function ξ_λ such that $\xi_\lambda(q, i)$ is exactly the costs user i has to pay if the mechanism selects users according to the characteristic function q. This function ξ_λ is budget balanced, that is, $\sum_{i=1}^{n} \xi_\lambda(q, i) = \mathrm{Cost}(q)$.

For a rate function $q : N \to \{0, 1, \ldots, \ell\}$ and $1 \leq k \leq \ell$, let $q_{=k} : N \to \{0, 1\}$ be the characteristic function of all users to whom rate ρ_k is assigned. Let $q_{\leq k}$ be the characteristic function of all users to whom one of the rates $\rho_1, \ldots, \rho_k$ is assigned. The fact that the rated mechanism should be composed of binary mechanisms manifests in the following two properties:

Layered Costs (LC): For all users i, $x_i = \sum_{\lambda=1}^{q_i} \xi_\lambda(q_{\leq\lambda}, i)$.

Split Session Costs (SSC): For all i, $x_i = \xi_{q_i}(q_{=q_i}, i)$.

Under the layered paradigm, the price x_i user i has to pay is exactly the sum of the first q_i cost-shares of i with respect to $\xi_1, \ldots, \xi_\ell$, since to get rate ρ_{q_i} user i has to receive the first q_i layers. Under the split session paradigm, the price is simply $\xi_{q_i}(q_{=q_i}, i)$, the share of i in the q_ith group.

4. RESULTS

We design a meta mechanism under the layered paradigm that uses a binary mechanism for each layer as a blackbox.

THEOREM 1. *If $\xi_1, \ldots, \xi_\ell$ are cross-monotonic and budget balanced, then there is a mechanism L that meets NPT, VP, CS, SP, BB, and LC. If each ξ_λ is only α_λ-BB, then L is only α-BB, where $\alpha = \max\{\alpha_1, \ldots, \alpha_\ell\}$.*

This meta mechanism is interesting on its own and can be applied to other pricing problems with rates. It remains an open question whether one can also achieve GSP for such a meta mechanism. Once we have this meta mechanism, we can plug various binary mechanisms into it. If the underlying multicast tree is fixed, we can for instance use the Shapley value (see e.g. [2]). If there is no underlying fixed multicast tree, then we can exploit the binary mechanism by Jain and Vazirani [4] to get a mechanism for the multicast problem with rates under the layered paradigm that meets NPT, VP, CS, SP, and BB. This mechanism works for general graphs and computes for each layer a multicast tree whose weight is at most twice the weight of an optimum Steiner tree, provided that the triangle inequality holds.

Then we show that for the split session paradigm, such a meta mechanism does not exist.

THEOREM 2. *There are cross-monotonic functions ξ_1, ξ_2 such that there is no mechanism for ξ_1, ξ_2 that meets NPT, VP, CS, SP, BB, and SSC.*

This insight complements nicely the results by Adler and Rubenstein that the split session paradigm is also harder than the layered paradigm in their setting.

Finally, we extend the techniques of Jain and Vazirani to a larger class of constrained forest problems by incorporating ideas of Goemans and Williamson [3]. This allows us to model extended multicast scenarios like having simultaneous (parallel) transmissions or several (mirrored) servers.

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